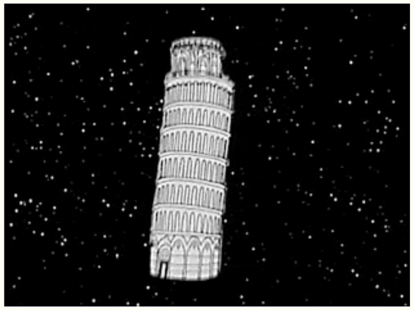
Falling Stuff



Do Falling Objects Accelerate?

- It sure seems like it!
  - Starts from rest, goes faster and faster....
- What about a feather, though?
  - Air resistance, drag
  - Terminal velocity
  - What if we could get rid of the air?
- What's responsible for the downwards force?
  - If it's accelerating, then a force is acting:

$$F = ma$$

Mass vs. Weight

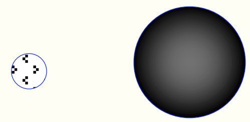
- <u>Mass</u> is reluctance to accelerate (mass ⇔ inertia)
- <u>Weight</u> is the *force* exerted by gravity
  - Go to the moon: Does your mass change?
    Does your weight change?

Acceleration Due to Gravity

- At the earth's surface, all objects experience the same acceleration from gravity!
  "g"= 9.8m/s^2 = 32 ft/s^2
- If the acceleration due to gravity is indeed universal, then...

Since $F = ma$, the gravitational force must be proportional to mass.

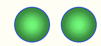
Golf Ball vs. Bowling Ball



- Which one is more massive?
- Which one experiences more gravitational force?
- Which one is most reluctant to accelerate?
- How do they respond to a gravitational force?

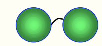
A Logical Argument

Consider two identical falling objects



Mass of each = M

Now imagine two more, connected together with a *tiny* thread

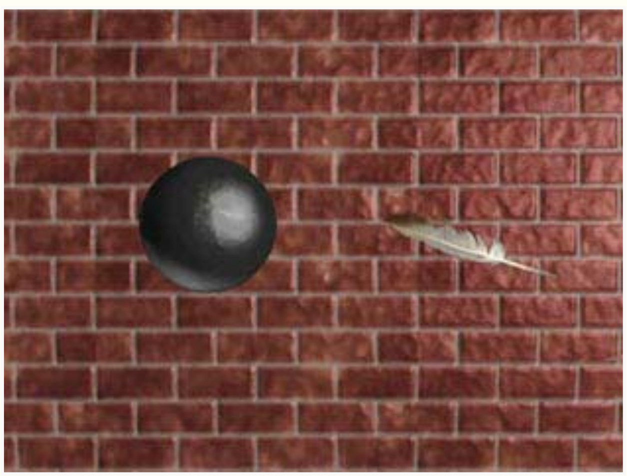


Total Mass = 2 M

But they all have the same acceleration!

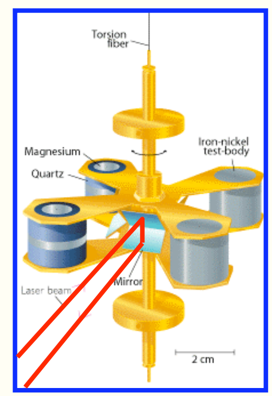
Force on the joined balls must be *twice* the force on one of them, since the mass doubled but the acceleration stayed the same.

Conclusion: Gravitational force must be proportional to mass



How Do We Know the Accelerations are the Same?

Experimental tests show the Universality of Free Fall is the same for different materials to within 0.00000000001%



Force Exerted by Gravity

If the gravitationally induced acceleration is the same for all objects at the surface of the Earth, then

Force exerted by gravity = (mass) × (acceleration due to gravity)

$F_{gravity} = m\, g$ = WEIGHT, where $g = 9.8$ m/s^2

For a mass of 100 kg, force from gravity at Earth's surface is

$F = 100$ kg × 9.8 m/s^2 = 980 Newtons.

(An apple weighs about 2 N, a golf ball weighs 1.4 N)

Said Another Way....

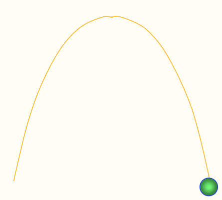
- Gravitational force is proportional to mass
- $\boldsymbol{F} = m\boldsymbol{a}$ gives an object's responding acceleration
- Divide both sides of the equation by "m"
- $\boldsymbol{a} = \boldsymbol{F}/m$
- Both numerator and denominator are proportional to "m", if force is gravity
- SO....acceleration is the same, regardless of the mass
- We'll return to this point when we consider General Relativity!

Falling Objects *Accelerate*

- Ignoring air resistance, falling objects near the surface of the Earth experience a constant acceleration of 9.8 m/s^2.
- That means if you drop something it goes faster and faster, increasing its speed downwards by 9.8 m/s in each passing second.

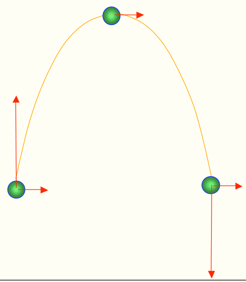
Gravitational Force is Acting All the Time!

- Consider a tossed ball.... Does gravity ever switch off?
- As a ball travels in an arc, does the gravitational force change?



An Example of the Reductionist Approach

- By breaking the motion into independent parts, analysis is simplified!
- The horizontal and vertical motions *are* independent

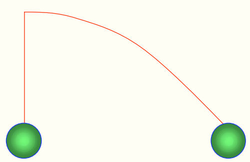


Components of Motion

- Break the motion into 2 aspects, “components”
  - Horizontal
  - Vertical
- Is there a force acting in the horizontal direction?
- Is there a force acting in the vertical direction?
- Does the ball accelerate in the horizontal direction?
  - Does its horizontal velocity change?
- Does the ball accelerate in the vertical direction?
  - Does its vertical velocity change?

Projectile Motion

- All objects released at the same time (with no *vertical* initial velocity) will hit the ground at the same time, regardless of their *horizontal* velocity
- The horizontal velocity remains constant throughout the motion (since there is no horizontal force)



Some Exercises

- A ball falls from rest for 4 seconds. Neglecting air resistance, during which of the 4 seconds does the ball’s speed increase the most?
- If you drop a ball from a height of 4.9 m, it will hit the ground 1 s later. If you fire a bullet exactly horizontally from a height of 4.9 m, it will also hit the ground exactly 1 s later. Explain.
- If a golf ball and a bowling ball (when dropped from the same height) will hit your foot at the same speed, why does one hurt more than the other?

Doing the Numbers

- Imagine dropping an object, and measuring how fast it's moving over consecutive 1 second intervals
- The vertical component of velocity is changing by 9.8 m/s in each second, downwards
- Let's approximate this acceleration as 10 m/s^2

Starting from rest, letting go:

Time Interval	Acceleration (m/s^2 down)	Vel. at end of interval (m/s down)
0 – 1 s	10	10
1 – 2 s	10	20
2 – 3 s	10	30
3 – 4 s	10	40
4 – 5 s	10	50

After an interval *t*, the velocity changes by an amount *at*, so that

$v_{final} = v_{initial} + at$

How fast was it going at the end of 3 sec?

$v_{initial}$ was 20 m/s after 2 sec

a was 10 m/s (as always)

t was 1 sec (interval)

v_{final} = 20 m/s + 10 m/s^2 × 1 s
= 30 m/s

Starting from rest, letting go:

Time Interval	Acceleration (m/s^2 down)	Init→Final Velocity (m/s down)	Average Velocity (m/s down)
0 – 1 s	10	0 → 10	5
1 – 2 s	10	10 → 20	15
2 – 3 s	10	20 → 30	25
3 – 4 s	10	30 → 40	35
4 – 5 s	10	40 → 50	45

The average velocity in the interval is just

$V_{avg} = ½(v_{initial} + v_{final})$

For the 1 – 2 s interval,

v_i = 10 m/s
v_f = 20 m/s

So v_{avg} = ½ (10+20) m/s
= 15 m/s

Starting from rest, letting go:

Time Interval	Acceleration (m/s^2 down)	Final Velocity (m/s down)	Average Velocity (m/s down)	Dist. moved (m down)	Final Position (m down)
0 – 1 s	10	10	5	5	5
1 – 2 s	10	20	15	15	20
2 – 3 s	10	30	25	25	45
3 – 4 s	10	40	35	35	80
4 – 5 s	10	50	45	45	125

This can all be done in shorthand

- The velocity at the end of an interval is just (the starting velocity) plus (the time interval times the acceleration):

$$\boldsymbol{v}_{\text{final}} = \boldsymbol{v}(t) = \boldsymbol{v}_{\text{init}} + \boldsymbol{a}t$$

- The position at the end of an interval is just (the starting position) plus (the time interval times the average velocity over the interval):

$$\boldsymbol{x}(t) = \boldsymbol{x}_{\text{init}} + \boldsymbol{v}_{\text{avg}}t$$

- Since $v_{\text{avg}} = \frac{1}{2}(v_{\text{init}} + v_{\text{final}})$, and $v_{\text{final}} = v_{\text{init}} + at$, $x(t) = x_{\text{init}} + \frac{1}{2}(v_{\text{init}} + v_{\text{init}} + at)\times t$, or

$$\boldsymbol{x}(t) = \boldsymbol{x}_{\text{init}} + \boldsymbol{v}_{\text{init}}t + \tfrac{1}{2}\boldsymbol{a}t^2$$

An aside on units and cancellation

What happens if you multiply an acceleration by a time?

Units of acceleration are m/s², units of time are s

Result is $\frac{\text{m}}{\text{s}^{\not 2}} \times \not\text{s} = \frac{\text{m}}{\not\text{s}\,\text{s}} \times \not\text{s} = \frac{\text{m}}{\text{s}}$

And this has units of velocity

Summary

- *Velocity* refers to both speed and direction
- *Acceleration* means a change in velocity
- *Mass* is a property of objects that represents their reluctance to accelerate
- If an object is accelerating, it's being acted on by an unbalanced force, and $\boldsymbol{F} = m\boldsymbol{a}$
- Gravity causes all objects to suffer the same acceleration, regardless of their mass or composition
- Gravitational acceleration only affects the vertical component of motion – think in terms of components

Assignments

- HW 2: due Friday (4/18):
  - Hewitt 11.E.16, 11.E.20, 11.E.32, 11.P.5, 2.E.6, 2.E.11, 2.E.14, 2.E.36, 2.E.38, 3.E.4, 3.E.5, 3.E.6, 3.E.19
  - turn in at lecture, or in box outside SERF 336 by 3PM
- Read Hewitt Chapters 2, 3, 4
  - suggested order/skipping detailed on website