The Human Machine

By no means exhaustive, this lecture looks at some aspects of the human body as a physics machine.

Mass Balance

Intake

We consume 2000–3000 kcal/day (male/female differences exist, in aggregate). At 4.2 kJ/kcal, this translates to 100–150 W of metabolic power. About 20 W goes to powering our brains (similar to a laptop).

As a rule of thumb, carbs yield 4 kcal/g; proteins also five about 4 kcal/g, and fats give about 9 kcal/g. A note on protein, though: it's hard to even get a nutrition expert to verify this, but if you use proteins for the amino acids to build your own structures (proteins, muscle), you *can't also* break it down for energy. Take your pick. If the body is not in need of proteins, but could use the energy, it may destroy the molecules for energy (in the liver). But you don't get it both ways. Somewhat similar to fat: you store it if you don't need it for energy at the time.

I found a handy formula that says for some random molecule (carb, protein, lipid), if its chemical formula is

$$\mathrm{C_cH_hO_oN_n},$$

then the energy per gram is

$$100\frac{c + 0.3h - 0.5o}{12c + h + 16o + 14n}\ \mathrm{kcal/g}.$$

The denominator is obviously the mass of the molecule in grams. The numerator tells us that we get the most per carbon atom, 30% as much from hydrogen, take a 50% hit for oxygen, and that nitrogen is energetially inert. It makes some sense. If the end product is CO_2, then CO_2 should offer no energy to us (put it in the formula). H_2O is likewise effectively neutralized in the formula. We can also work through unit conversions and ascertain that each carbon atom provides 4 eV of energy (a sensible scale), while each hydrogen provides 1.3 eV and each oxygen deducts 2 eV. Let's make a table for some examples!

formula	kcal/g	% C	% H	what is it?
$C_6H_{12}O_6$	3.7	40	7	glucose
$C_5H_{10}O_3N_2$	4.4	41	7	amino acid
$C_{58}H_{112}O_6$	9.8	77	12	fat

Definetly the right ballpark. If we assume a 2000 kcal daily diet in which 30% of the kcal comes from fat, we find that we need to eat 420 g of the stuff that matters, energetically, which is about a pound of food. This does not count fiber and water content of food.

Output

In terms of mass output, we consider such things gross. But thinking like a physicist, we just care about masses. The mass arrives through a single port, but leaves through many (more than you might think!). In steady state (approximately true even during periods of gain/loss), **mass-in equals mass-out**. That seems so obvious it doesn't need to be stated—let alone bolded. But lots of people (who for instance would like to lose weight) don't seem to get it (they'll argue: their body is different; is physics not the same?).

Let's say on a typical day a person consumes 1 kg of food (half of which is energetic content, the other half water and fiber), and 1 kg of drink (1 L). Let's further estimate that in a typical day the person dumps

0.25 kg of solid waste and 1 L of number-one. We have another 0.75 kg unaccounted for. Where does this go? Breathe, Neo.

That's right. Breathing is more important to mass balance than you may have guessed. If we eat 420 g of metabolically useful food in a day, about 220 g is in the form of carbon ($0.3 \times 0.77 + 0.7 \times 0.4$ times 420 g for a 30% fat diet; see table), and 36 g in hydrogen. This turns into about 800 g of CO_2 and 320 g of H_2O after "combustion." We breathe out the CO_2, thereby losing the 220 g of mass injested in food-carbon. That's a half-pound per day just by breathing.

We also breathe out air saturated with water (∼100% humid). Two things are relevant here. Generally, the air we breathe out is hotter than the air we breathe in, so can hold more moisture. Also, the air we breathe in is not (we ho[e) 100% humid. Saturation vapor pressure of water at 37 C (body temperature) is 45 Torr, while air at 20 C is 17 Torr. If we exhale 1 L every 10 seconds, that's 8640 L/day. Multiply by the partial pressure of water: 45/760, and we therefore breathe out 500 L/day of water vapor. At approximately 20 L/mol, we're dealing with 25 moles of water, coming to 450 g. But we must deduct water we *inhale*. This is highly variable, and dependent on conditions. But for example, at 20 C and 50% humidity, the 8640 L we breathe in contains 100 L of water vapor, or 5 moles, or 90 g. So we're left with 360 g of net exhaled water.

We have therefore accounted for 600 g of our 750 g shortfall. While the 750 g was a crude ballpark estimate, we do have one other channel we might consider: perspiration. It's easy to amass a few hundred grams of perspiration in a day (much of it does not bead up and you may not even be aware it's happening). A heavy day may produce over a liter, and we generally find ourselves drinking more to keep up.

Finally, it is worth a bit of tallying on water balance. As a crude guess, we may imbibe 1 L, and also pee 1 L. We calculated that we convert hydrogen in our food to about 320 g of hater each day, and breathe out about that same amount. Lastly, some of our food has water in it, and we have the perspiration channel potentially balancing the books on that one. Naturally, individual water molecules don't keep track of how they came in and how they leave, but it all seems to add up to sensible amounts.

The Physics Diet

Having explored some of the factors in mass balance, we are now ready for the Tom Murphy approved diet plan. If you want to lose weight, it's actually painfully simple. Put less food into your mouth, and breathe plenty. The latter is nicely accomplished with aerobic exercise. It's just mass conservation. What could possibly make more sense?! I don't want to hear about your problems. If you are in control of the route into your mouth (usually via your hand, under your control), then it's technically easy to change.

Mechanical Output

Muscles are somewhere around 18–26% efficient at turning metabolic energy into mechanical output. I have traditionally used 25%, but perhaps this is on the optimistic end and 20% is just as reasonable. So if a 70 kg person climbs 1000 m (a substantial mountain climb), it requires 700 kJ or 170 kcal of output, thereby demading 700–800 kcal of food to be et. A hard day (backpacking, biking, digging, playing hard) might result in your eating twice the amount of food as normal, meaning that you accomplished 400–500 kcal of external work (if consuming an extra 2000 kcal). If the activity stretched over 8 hours, we're talking 50–60 kcal/hr. To get Watts, we multiply by 4184/3600 (nearly unity) to get 60–70 W of output, or about 0.1 horsepower.

This puts things in perspective a bit. Maintaining 100 W output for much of the day is very hard, and would require more than doubling your normal food intake. The best athletes might maintain 200 W for hours on end, and can sustain 300–400 W for about a half-hour.

Testing Yourself

One of the homework problems will have you perform a real experiment (get out of the armchair!) to see how much power you can put out for a bit. Here is an example for myself, using concepts from earlier in the class. When I bike up Gilman Drive on a standard mountain bike (no electric assist, though I do usually commute on such a thing), I go from the bottom of the hill (at the freeway and bike trail) to the pedestrian bridge at the top of the hill near Mayer Hall in almost exactly 10 minutes. Google Earth tells me it's 3.0 km and an elevation gain of 68 m (from 50 m to 118 m above sea level). My base mass is 80 kg, plus 15 kg of bike, and another 5 kg for clothes and gear, making a tidy 100 kg. The average speed is 5.0 m/s (I'm really killing it on convenient numbers!). Air drag is $\frac{1}{2}c_D\rho Av^2$. We'll use 0.6 m^2 for c_DA, and 1.25 kg/m^3 for ρ, resulting in 9.4 N of force. Multiply by a distance of 3 km and get 28 kJ fighting air. Rolling resistance is probably about 0.005 times the normal force, or 5 N. That makes 15 kJ over 3 km. And the easy part (to calculate!): climbing against gravity requires 68 kJ of energy on this hill. The total is 111 kJ, dominated by the climb (not surprisingly). In 600 s, this constitutes an average output power of 185 W. Almost athletic level! 111 kJ is 26 kcal, requiring me to eat 4–5 times this much, or about 120 kcal to replenish.

Walking

Walking is a pretty efficient exercise (not as efficient as biking, for distance covered, though). Estimates are that it takes about 50 W to walk.

We can model walking as having pendulum legs. But these are not lead-foot pendula with all the mass at the foot. We model the leg as having uniform mass per unit length (not quite right), mass m, length L, moving through angle, θ. The potential energy is $U = mgL\theta^2/4$ (mass midpoint at $L/2$ and height change this distance times $\theta^2/2$). Kinetic energy is $T = \frac{1}{2}I\dot{\theta}^2 = \frac{1}{6}mL^2\dot{\theta}^2$. We combine these into a Lagrangian $\mathcal{L} = T - U$ and apply the Eulter-Lagrange equation to build our differential equation:

$$\frac{1}{3}L\ddot{\theta} = -\frac{1}{2}g\theta.$$

We can therefore identify the frequency as

$$\omega = \sqrt{\frac{3}{2}\frac{g}{L}}.$$

The full period is $\tau = 2\pi/\omega$, and if the swing is through an angle $\theta_{\max} \sim 30°$, the stride (one step) corresponds to a distance $2\theta L \approx L$ (about right, if you measure stride and leg length).

Walking velocity is then this length divided by half the period, or $v \approx 2L/\tau = L\omega/\pi = \frac{1}{\pi}\sqrt{3gL/2}$. If leg length is $L = 0.8$ m, we get a velocity of 1.1 m/s, or 2.4 m.p.h. (4 kph), which strikes me as very plausible.

Jumping

How high can we expect to jump? Let's assume this is power-limited: we can only burst at so much power. I might guess 1000 W for the second or less we spend developing the jump. This is related to the amount of muscle energy we store and the rate at which we can access it. More muscles mean more energy available in the time interval of interest.

From a partial crouch position, we might develop the jump over half our leg length, $L/2$. Assuming constant force, we achieve constant acceleration, a, so that $L/2 = \frac{1}{2}at^2$, where t is the amount of time over which we execute the leg extension. So $t^2 = L/a$. We achieve a velocity $v_f = at = \sqrt{La}$. The kinetic energy achieved at the end is $T = \frac{1}{2}mv_f^2 = \frac{1}{2}mLa$ in time $t = \sqrt{L/a}$ so that we exert a power

$$P = \frac{1}{2}\frac{mLa}{\sqrt{L/a}} = \frac{1}{2}m\sqrt{La^3}.$$

We can solve now for acceleration given a power limit of 1000 W and then express the height via $mgh = \frac{1}{2}mLa$ (potential equals kinetic) by which we conclude

$$h = \frac{1}{2}\frac{La}{g} = \frac{1}{2g}\left(\frac{2PL}{m}\right)^{\frac{2}{3}}.$$

Inserting $P = 1000$ W, $m = 80$ kg, $L = 0.8$ m yields a height of 0.4 m. Try it yourself. You'd be surprised how pathetic a human jump is. I know for me, I can reach about 8 feet, 4 inches from a standing position, and can *maybe* (still) jump high enough to get about 4 inches over the rim of a basketball hoop, meaning I jump 2 ft, or 0.6 m. I have longer legs, so maybe I can peak at 1800 W! Indeed, this is more in line with an experiment bolting up stairs.

Other animals have similar energy density in their muscles, so stored energy is proportional to mass. If the amount of stored energy available for immediate release in a jump is also proportional to mass, then we would expect the jump height to be fairly consistent across the animal kingdom. Indeed, a flea can jump a similar height as a human.