

## Course Structure and Goals

This course is part of a set of three by Fuller, Diamond, and Murphy. Each one brings a different perspective and focus:

Fuller:	estimation of quantum/microscopic physics & application to large systems
Diamond:	scaling relationships; fluids, turbulence, waves & wave fronts
Murphy:	quantitative estimates of common phenomena & cultural/societal scale problems

You will learn to forage on raw, undigested problems where **you** generate the content and inputs! Say goodbye to pre-packaged twinkies; learn to survive on hunting/gathering techniques. Why? This is how many talented physicists think/operate (Feynman, Fermi, other famous Fs).

## A Note on Numerics

IMHO, there are certain numbers (with units) a physicist—especially an experimentalist—needs to know. I carry perhaps an abnormal amount (sometimes to ridiculous precision), but *they serve me well*. Knowing  $c$ ,  $k$ ,  $e$ ,  $h$ ,  $m_e$ ,  $m_p$ ,  $\sigma$ ,  $G$ ,  $\mu_0$ ,  $\epsilon_0$ ,  $R_\oplus$ ,  $M_\oplus$ ,  $r_{\text{AU}}$ ,  $N_A$ , various densities, assorted nucleon mass numbers, materials properties, loads of solar system and cosmological numbers gives me tremendous freedom. I know them because I *use* them: not the result of a memorization campaign.

For much of this class,  $\pi = 3 = \sqrt{10}$ . 2 and 3 *may* be considered distinct, but 8 and 9 often will not be considered different enough to care. Sometimes I will insert unnecessary digits in numbers only to later ignore them. But in doing rough math, it helps to keep track of rounding slights so that we might balance them in a later rounding operation—or at least not exacerbate with cumulative transgressions in the same direction.

## Famous Fermi Problem: How many piano tuners in Chicago?

Draw on observation, awareness, common experience, guesses. The more guesswork that goes into the mix, the less likely the result is *biased* in the end—at the expense of imprecision. This is because overestimates are compensated by underestimates. The central limit theorem applies.

One approach to the problem:

- How many people in Chicago area?
- How many households does that make?
- What fraction of households have a piano?
- How long does a piano go between tunings?
- What fraction of piano tuners *care* about out-of-tune piano enough to pay?
- How long does it take to drive to house, tune piano, drive back?
- How many hours per week does a piano tuner likely work?

Try it out, then check the phone book. You're likely within a factor of a few.

Now figure out how much the piano tuner charges for the service.

## Example Problems

Examples abound: how many molecules did you just breathe in that were part of Julius Caesar's last gasp?; how far must a car travel to wear a one-molecule layer off a car tire?; how many laser pointers would we need to point at the Moon to make a visible spot?; how long would it take for air to escape from a bullet hole in an airplane at altitude?; by what factor might pedestrians outweigh traffic on a bridge, if crowded with people? These questions share the property of being very easy/straightforward to ask. You provide the numerical inputs yourself. Let's dissect another example.

### Kids laughing uncontrollably

*Right now*, how many kids are laughing *so hard* that milk (or cultural equivalent) is coming out of their noses?

- How many in this room have experienced this personally (what fraction)?
- How long does the episode last? (in seconds; not Planck units)
- How many seconds old are kids when they outgrow this threat? ( $1 \text{ yr} = \pi \times 10^7 \text{ sec}$ )
  - what fraction of a typical kid's time is spent spewing milk through nose?
- How many kids are there in the world?

Now how far off might we be in our estimate? What answers would we get if we pushed the guesses to the comfortable extremes?

Notice a *pattern* in these Fermi problems: while the original question is short and well-enough posed, the first step is to ask loads more **questions**. In this way, Fermi problems mirror how science is done. *Learning to ask the right/relevant questions is central to the art of science.*

### Does it feel hot in here to you?

How much warmer will a 500-seat auditorium be after 2 hours at full capacity?

1. How large is the room?
  - (a) guess based on intuition:  $20 \times 40 \text{ m}^2$ , 6 m high  $\rightarrow 5 \times 10^3 \text{ m}^3$
  - (b) seat area is 0.6 m wide, 1.0 m deep  $\times$  factor 1.10 for aisles  $\times$  factor 1.2 for stage  $\rightarrow 400 \text{ m}^2$  floor area times 6 m high  $\rightarrow 2.5 \times 10^3 \text{ m}^3$
2. How much mass is being heated?
  - (a) air:  $\frac{3}{4} \text{ N}_2$  at 28 g/mol  
 $\frac{1}{4} \text{ O}_2$  at 32 g/mol  $\rightarrow 29 \text{ g/mol}$  average  
In STP, 1 mole occupies 22.4 liters =  $0.0224 \text{ m}^3$   
1  $\text{m}^3$  of air has  $\frac{0.029 \text{ kg/mol}}{0.0224 \text{ m}^3/\text{mol}} = 1.3 \text{ kg/m}^3 \rightarrow 3 \times 10^3 \text{ kg}$  in room, using 1(b)
  - (b) seats: 10 kg per person  $\rightarrow 5 \times 10^3 \text{ kg}$
  - (c) walls and floor and ceiling, to partial depth  
 $A = 2 \times 400 \text{ m}^2 + 4 \times (6 \text{ m})(20 \text{ m}) \approx 1500 \text{ m}^2$   
if heated 1 cm thick  $\rightarrow 15 \text{ m}^3$  of material  
at average density of water  $\rightarrow 15 \times 10^3 \text{ kg}$
3. Heat capacity?

- (a) metals: 500 J/kg/K
- (b) wood, air, most stuff: 1000 J/kg/K → use when in doubt!
- (c) liquids: 2000 J/kg/K
- (d) water: 4184 J/kg/K
- (e) Aside:  $\frac{3}{2}kT$  energy per atom/particle →  $\frac{3}{2}k$  J/K per atom →  $\frac{1000N_A}{A}$  atoms/kg, where  $N_A$  is Avogadro's number ( $6 \times 10^{23}$ ) and  $A$  is atomic number. So specific heat is approximately

$$c_p \approx \frac{1500N_A k}{A} \sim \frac{12000}{A} \text{ J/kg/K}$$

much of our world is constructed from  $A \sim 10\text{--}50$  atoms/molecules, thus 200–1000 J/kg/K is typical

#### 4. Power input?

- (a) human diet  $\sim 2000$  Cal/day →  $2 \times 10^6$  cal/day →  $8 \times 10^6$  J/day at 4.184 J/cal
- (b)  $(60)(60)(24) = 86400$  seconds in day →  $\sim 100$  W average power output per person
- (c) energy input in 2 hours is  $100 \text{ W} \times 7200 \text{ s} \rightarrow 7 \times 10^5 \text{ J}$  per person;  $3.5 \times 10^8 \text{ J}$  total

#### 5. Putting together:

- (a) heats air alone by 100 K
- (b) heats seats alone by 70 K
- (c) heats walls, etc. alone by 20 K
- (d) in combination, these numbers don't add, because heat paths in parallel:  
all together:  $\frac{3.5 \times 10^8 \text{ J}}{23 \times 10^3 \text{ kg} \cdot 1000 \text{ J/kg/K}} \approx 15 \text{ K}$

#### 6. Caveats:

Less hot if thermal penetration of walls/floor/ceiling exceeds  $\sim 1$  cm  
 Less hot if heat escapes through walls/ceiling  
 Less hot if there is *any* ventilation. *Please* let there be some ventilation!

#### 7. Follow-up questions

- (a) How much airflow is required to limit  $\Delta T$  to  $< 3^\circ\text{C}$ ?
- (b) If fans eject air at 15 m/s, how much fan area is required to satisfy this? [I get about  $2 \text{ m}^2$ ]

### Vaporized on impact

From at least how high must a rock be dropped if it is to vaporize on impact?

Two methods: A) guess at temperature of vaporization & use heat capacity; B) guess at bond strength in eV.

(A) Metals and rocks are probably similar: melt around 1500–3000 K ( $^\circ\text{K} \approx ^\circ\text{C}$  for these high values). Would guess they vaporize slightly higher temperature, like 2000–4000 K. At 1000 J/kg/K, we need  $2\text{--}4 \times 10^6$  J/kg of energy input. Hold that thought...

(B) Guess 5 eV per bond to remove atom from lattice →  $8 \times 10^{-19}$  J. Typical “rock atom” (Rk?) has molecular weight  $\sim 30$  g/mol (silicon). To remove (vaporize) one mole of atoms requires  $6 \times 10^{23} \times 8 \times 10^{-19} \text{ J} \approx 5 \times 10^5 \text{ J}$ , so that 1 kg ( $\sim 30$  moles) requires  $1.5 \times 10^7 \text{ J/kg}$ . This is 4–7 times bigger than result of (A).

This disagreement illustrates the value of two (ore more) angles of attack. Agreement can bolster confidence in the approach. Disagreement can expose a misunderstanding about the problem; provide a sense of the uncertainty or range of possibilities; and can act to hone intuition about the physics.

For now, we split the difference. Want some justification? Rock probably has a higher vaporization temperature than metals (think ceramic; Hg). The guess of 5 eV may be too high—was just a guess. The geometric mean is about  $7 \times 10^6$  J/kg.

The energy per kg in gravitational potential energy is just  $gh$ , so  $h = 7 \times 10^6/g \approx 7 \times 10^5$  m, or about one tenth the earth radius. Since the value of  $R_{\oplus} + h$  differs from  $R_{\oplus}$  by only 10%, we do not need to use  $GMm/R$  in computing the potential difference [ $g$  is constant enough over the interval to suffice for an order-of-magnitude estimate].

Note that we assume all the energy goes into the rock, and not the atmosphere (very thin) or the ground. But we put a meaningful scale on the problem.

## Checking outrageous claims with physics

Urban legends, “Darwin awards,” movie “physics” should all be checked with numbers.

Example: Man lifts self & lawn chair & six pack beer & gun into LAX approach path at 10,000 feet.

How much helium would this require?

Mass: man: 70 kg; chair and gun: 8 kg; beer: 2 kg; beer gut: 10 kg  $\rightarrow$  90 kg

Any gas at STP occupies 22.4  $\ell$ /mol. At 29 g/mol, air  $\rightarrow$  1.3 kg/m<sup>3</sup>

He is 4 g/mol, so replace volume of 29 g/mol with 4 g/mol and get effective  $-25$  g/mol of buoyant molecular weight  $\rightarrow -1.1$  kg/m<sup>3</sup>. So to offset 90 kg of “downward” mass, we need about 80 m<sup>3</sup> of He volume. This is BIG! Over 5 meters diameter! This is not casually built up by assembling small balloons together.

But it gets worse. To be neutral at 10,000 feet  $\sim$  3 km, must offset 90 kg of air *at a reduced density*. Atmospheric density is exponential with a scale height,  $\lambda \sim 7$  km, so about  $\frac{2}{3}$  surface density. So now air density is 0.9 kg/m<sup>3</sup>, and He displaces  $-0.7$  kg/m<sup>3</sup>. So 90 kg of displacement requires 120 m<sup>3</sup>, or about 50% more than needed for neutral buoyancy at ground level.

If someone takes the time to gather enough helium to float around the neighborhood drinking beer, do you really think they overshoot by 50%? Why not sit in chair and test iteratively as balloons are added? I guess if you’re not a theorist *or* an experimentalist, you’re just a moron.

As an aside, a 50% overshoot in buoyancy means the initial upward acceleration will be  $0.5g$ ! But the behemoth balloon will have significant air resistance, so may hit terminal velocity fast. We can explore this! Terminal velocity is reached when the drag force equals the motive force (usually  $mg$  for falling stuff, in this case it’s  $0.5g$  of buoyant force).  $F_{\text{drag}} = \frac{1}{2}c_D A \rho_{\text{air}} v^2$ , where  $c_D$  is the coefficient of drag (about 1 for most shapes), and  $A$  is cross-sectional area presented to flow. Setting this to equal  $0.5g$  (one-halves cancel):  $v^2 = \frac{900\text{N}}{30\text{m}^2 \cdot 1.3\text{kg/m}^3} \approx 25\text{m}^2/\text{s}^2 \rightarrow v \approx 5\text{m/s}$ . This is reached after about one second at  $0.5g$ . So the moron would feel an initial jerk followed by a smooth ascent.

**Now go crazy with your own explorations!**

## Recommended Books

*Guesstimation*, by Lawrence Weinstein and John A. Adam; many example problems; light and fun

*Flying Circus of Physics*, by Jearl Walker; food for thought on complex everyday physics